13) $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{bmatrix}$

$$P(\lambda) = \det\left(\begin{bmatrix} \lambda-1 & 0 & 0 \\ -2 & \lambda-1 & 2 \\ -3 & -2 & \lambda-1 \end{bmatrix}\right) = (\lambda-1).(\lambda^2-2\lambda+5) = \lambda^3-2\lambda^2+5\lambda-\lambda^2+2\lambda-5 =$$
$$= \lambda^3-3\lambda^2+7\lambda-5.$$

Autoval. $\rightarrow P(\lambda)=0 \rightarrow$ $\lambda_1 = 1$, $\lambda_2 = 1+2i$, $\lambda_3 = 1-2i$

Para $\lambda=1$

$$\begin{pmatrix} 0 & 0 & 0 \\ -2 & 0 & 2 \\ -3 & -2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} -3 & -2 & 0 \\ -2 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow F2 \rightarrow 2F1-3F2 \begin{pmatrix} -3 & -2 & 0 \\ 0 & -4 & -6 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\rightarrow \begin{cases} -3x-2y=0 \rightarrow -3x = -\frac{6}{2}z \rightarrow -3x=-3z \rightarrow x=z. \\ -4y-6z=0 \rightarrow y = -\frac{3}{2}z \end{cases}$$

$$\rightarrow \bar{X} = \left(z; -\tfrac{3}{2}z; z\right) = z.\left(1; -\tfrac{3}{2}; 1\right)$$

AUTOVECTOR (MÚLTIPLO)

$\lambda=1 \rightarrow (2;-3;2)$

Para $1+2i=\lambda$

$$\begin{pmatrix} 2i & 0 & 0 \\ -2 & 2i & 2 \\ -3 & -2 & 2i \end{pmatrix} \rightarrow \begin{pmatrix} -3 & -2 & 2i \\ -2 & 2i & 2 \\ 2i & 0 & 0 \end{pmatrix} \begin{matrix} F2 \rightarrow 2F1-3F2 \\ F3 \rightarrow 2iF1+3F3 \end{matrix} \begin{pmatrix} -3 & -2 & 2i \\ 0 & -4-6i & -6+4i \\ 0 & -4i & -4 \end{pmatrix}$$

$\rightarrow$ $\left.\begin{array}{l} x=0 \\ y=iz \end{array}\right\}$ SISTEMA RESUELTO POR COMPU.

$\rightarrow$ $\bar{X} = (0, iz, z) = z \cdot \underbrace{(0, i, 1)}_{\substack{\text{AUTOVECT.} \\ \lambda = 1+2i}}$

Para $\lambda = 1-2i$ será el conjugado:

autovector $\lambda = 1-2i \rightarrow (0, -i, 1)$

$$Y_1 = e^t \cdot \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix} \qquad Y_2 = e^{(1+2i)t} \cdot \begin{pmatrix} 0 \\ i \\ 1 \end{pmatrix} \qquad Y_3 = e^{(1-2i)t} \cdot \begin{pmatrix} 0 \\ -i \\ 1 \end{pmatrix}$$

$\rightarrow$ $Yt = (Y_1, Y_2, Y_3)$

Se podría pasar todo a reales.